Lower Fractional Designs

BIOE 498/598 PJ

Spring 2022

Review

- A full factorial design with k factors requires 2^k runs.
- A half factorial design uses only 2^{k-1} runs.
  - Begin with a base design.
  - Set the remaining factor equal to an interaction (generator, $E = AB$)
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- Half-fractional factorials in action
- Today we define quarter- or eighth-factorial designs!

A large (2^5) unreplicated study

```
tumor <- read.csv("TumorInhibition.csv")
head(tumor)
```

```
##    A  B  C  D  E inhibition run half pb
## 1 -1 -1 -1 -1 -1         61   1    0  1
## 2  1 -1 -1 -1 -1         53   2    1  0
## 3 -1  1 -1 -1 -1         63   3    1  1
## 4  1  1 -1 -1 -1         61   4    0  0
## 5 -1 -1  1 -1 -1         53   5    1  0
## 6  1 -1  1 -1 -1         56   6    0  1
```

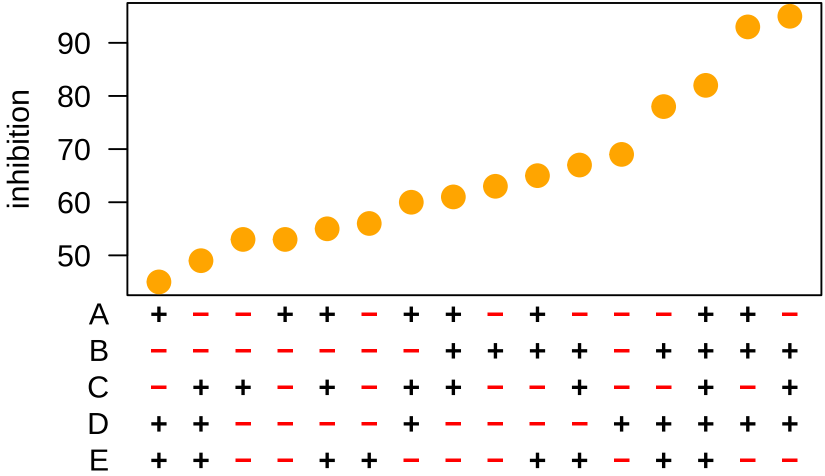
Let's pretend it was a 2^{5-1} half-fractional

```r
half_tumor <- tumor[tumor$half==1, ]
half_tumor
```

```
##       A  B  C  D  E inhibition run half pb
## 2     1 -1 -1 -1 -1         53   2    1  0
## 3    -1  1 -1 -1 -1         63   3    1  1
## 5    -1 -1  1 -1 -1         53   5    1  0
## 8     1  1  1 -1 -1         61   8    1  0
## 9    -1 -1 -1  1 -1         69   9    1  0
## 12    1  1 -1  1 -1         93  12    1  1
## 14    1 -1  1  1 -1         60  14    1  1
## 15   -1  1  1  1 -1         95  15    1  1
## 17   -1 -1 -1 -1  1         56  17    1  0
## 20    1  1 -1 -1  1         65  20    1  0
## 22    1 -1  1 -1  1         55  22    1  0
## 23   -1  1  1 -1  1         67  23    1  1
## 26    1 -1 -1  1  1         45  26    1  0
## 27   -1  1 -1  1  1         78  27    1  0
## 29   -1 -1  1  1  1         49  29    1  1
## 32    1  1  1  1  1         82  32    1  0
```

Step 2: Look at the data

```
farplot(half_tumor, response="inhibition", factors=c("A","B","C","D","E
```



Step 3: Fit a linear model

```r
half_model <- lm(inhibition ~ A*B*C*D*E, data=half_tumor)
```

```r
show_effects(half_model, scaling=2,
```

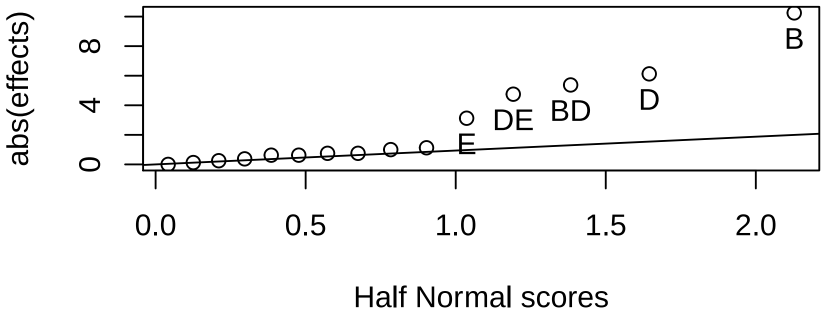
```
##          B     20.5
##          D     12.25
##        B:D     10.75
##        D:E     -9.5
##          E     -6.25
##        C:E      2.25
##          A     -2.
##        A:B      1.5
##        B:C      1.5
##        A:E      1.25
##        B:E      1.25
##        A:D      -.75
##        A:C       .5
##        C:D       .25
##          C       .
##      A:B:C     NA.
##      A:B:D     NA.
##      A:C:D     NA.
```

```r
show_effects(model, scaling=2, inter18)
```

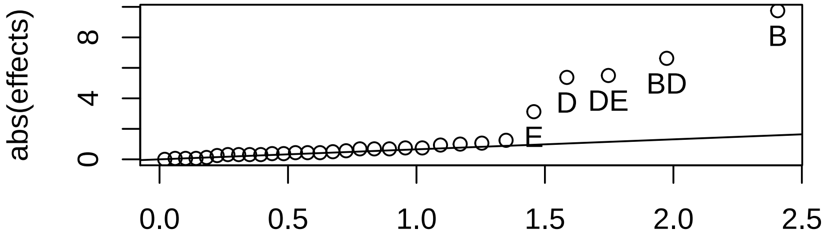
```
##          B     19.5
##        B:D     13.25
##        D:E    -11.
##          D     10.75
##          E     -6.25
##      A:C:E     -2.5
##        C:D      2.125
##        B:E      2.
##      A:B:E     -1.875
##      A:B:C      1.5
##    A:B:C:E      1.5
##      A:B:D      1.375
##        A:B      1.375
##          A     -1.375
##      B:C:D      1.125
##    A:C:D:E      1.
##        A:D      -.875
##        B:C       .875
```

Visualizing with half-normal plots

```
daewr::halfnorm(na.omit(get_effects(half_model)))
```




```
daewr::halfnorm(get_effects(model))
```



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$$B + AD + ABCE + CDE$$

$$C + ABCD + AE + BDE$$

$$D + AB + ACDE + BCE$$

$$E + ABDE + AC + BCD$$

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$$D + AB + ACDE + BCE$$

$$E + ABDE + AC + BCD$$

$$BC + ACD + ABE + DE$$

$$BE + ADE + ABC + CD$$

Eighth fractional design: 2^{6-3}

Factors A, B, C, $D = AB$, $E = AC$, $F = BC$

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Factors A, B, C, $D = AB$, $E = AC$, $F = BC$

$$I = ABD = ACE = BCF$$

Also, all combinations:

$$\begin{aligned}
I^2 &= I = (ABD)(ACE) = BCDE \\
I^2 &= I = (ABD)(BCF) = ACDF \\
I^2 &= I = (ACE)(BCF) = ABEF \\
I^3 &= I = (ABD)(ACE)(BCF) = DEF
\end{aligned}$$

Defining relation:

$$I = ABD = ACE = BCF = BCDE = ACDF = ABEF = DEF$$

Which generator should I choose?

A generator's optimality is assessed with three criteria:

- **Resolution:** difference in the level of confounding.
- **Aberration:** the multiplicity of the worst confounding.
- **Clarity:** # of confounded main effects or two-way interactions.

Criterion #1: Design Resolution

The resolution of a fractional design is the length of the shortest word in the defining relation.

For the 2^{5-2} design generated by D=AB and E=AC, the definig relation is

$$I = ABD = ACE = BCDE$$

This is a Resolution III design. (Resolution is written with Roman numerals.)

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- Main effects ($i = 1$) are confounded with secondary ($3 - 1 = 2$) interactions.

Resolution IV

- Main effects ($i = 1$) are confounded with tertiary ($4 - 1 = 3$) interactions.
- TWIs ($i = 2$) are confounded with other TWIs ($4 - 2 = 2$).

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A design with resolution R contains a full factorial design for any subset of $k = R - 1$ factors.

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If after the fractional experiments you drop to k factors you can re-analyze the data for all the interactions.

Criterion #2: Design Aberration

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- **Aberration:** Number of words with length equal to the resolution.

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$$I = ABCDF = ABCEG = DEFG \quad \text{resolution IV, aberration 1}$$

$$I = ABCF = ADEG = BCDEFG \quad \text{resolution IV, aberration 2}$$

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- **Resolution:** Length of the shortest word in the defining relation.
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$$I = ABCDF = ABCEG = DEFG \quad \text{resolution IV, aberration 1}$$

$$I = ABCF = ADEG = BCDEFG \quad \text{resolution IV, aberration 2}$$

We favor the design with the lower aberration. It will have fewer main effects confounded with low-order interactions.

Criterion #3: Clear Effects

A main effect or two-way interaction effect is **clear** if it is only confounded with higher order terms (three-way or higher).

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Clear effects always lead to tradeoffs. For a 2^{6-2} design:

$I = ABCE = ABDF = CDEF$	6 main effects clear
$I = ABE = ACDF = BCDEF$	3 main effects + 6 TWIs clear

Overall design guidelines

1. Choose the highest **resolution** that fits your budget.
2. For that resolution, choose the **minimum aberration** design.
3. If you have particular effects that you know are signficant, try to choose a factor or generator that clears them.