Alternative Fractional Factorial Designs

BIOE 498/598 PJ

Spring 2022
The efficiency of fractional factorial designs offsets the exponential increase in runs for factorial designs.
How low can we go? (zoomed in)
<table>
<thead>
<tr>
<th>number of factors</th>
<th>8</th>
<th>16</th>
<th>32</th>
<th>64</th>
<th>128</th>
<th>256</th>
<th>512</th>
<th>1024</th>
<th>2048</th>
<th>4096</th>
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<tbody>
<tr>
<td>Resolution III</td>
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<td>Resolution IV</td>
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<td>Resolution V</td>
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<td>Resolution VI</td>
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<td>First design is</td>
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</table>

- Resolution III up to factors: 31, 63, 127
- Resolution IV up to factors: 32, 64, 80, 160
- Resolution V up to number of factors: 33, 47, 65
- Resolution VI up to number of factors: 24, 34, 48
- First design is MA up to number of factors: 31, 63, 127, 36, 29, 28, 32, 26
Foldover Designs

Imagine a $2^{6-3}_{III}$ design with

$$D = AB, \quad E = AC, \quad F = BC$$

$$I = ABD = ACE = BCF = DEF = BCDE = ACDF = ABEF$$

After analysis, we find that both $B$ and $D$ are significant.

Since $D = AB$, the significance of $D$ might be due to $B$ and $AB$.

We can *augment* the design by doubling the runs with $D$ flipped. This clears $D$ and its interactions.

<table>
<thead>
<tr>
<th>Run</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>+</td>
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<td>+</td>
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<tr>
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<td>+</td>
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<tr>
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</tbody>
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Mirror image designs

If we combine a Resolution III design with its mirror image (all factors flipped), we have a Resolution IV design with all main effects clear.

If we add a blocking factor we can perform the experimental batches sequentially.

As with foldover designs, mirror image designs are only necessary if more than one main effect is significant.
Sometimes logistics force us to group runs into “blocks”.

Examples

- Mice need to be housed in separate cages.
- The experimenter cannot do all runs in a single day/batch.
- Two experimenters need to split up the runs.
- A single container of reagent doesn’t cover the experiment.
Sometimes logistics force us to group runs into “blocks”.

Examples

- Mice need to be housed in separate cages.
- The experimenter cannot do all runs in a single day/batch.
- Two experimenters need to split up the runs.
- A single container of reagent doesn’t cover the experiment.

A “blocking factor” is added to these experiment to capture inter-block differences.

Blocks are added to designs and analyzed like any other factor.

Most block × factor interactions are ignored.
Blocking our Pilot Plant Experiment

```r
pilot <- read.csv("PilotPlantBlocked.csv")
farplot(pilot, "yield", c("T","C","K","block"))
```
Building our normal model

```r
model <- lm(yield ~ T*K*C, data=pilot); show_effects(model)
```

```r
## (Intercept) 60.25
## T 9.75
## K -.5
## C -1.
## T:K 3.5
## T:C 2.
## K:C 1.75
## T:K:C 4.25
```

```r
model <- lm(yield ~ T*K*C + block, data=pilot); show_effects(model)
```

```r
## (Intercept) 60.25
## T 9.75
## K -.5
## C -1.
## block 4.25
## T:K 3.5
## T:C 2.
## K:C 1.75
## T:K:C NA.
```
Building our normal model

```r
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## T:K  3.5
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model <- lm(yield ~ T*K*C + block, data=pilot); show_effects(model)
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## T  9.75
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## C -1.
## block  4.25
## T:K  3.5
## T:C  2.
## K:C  1.75
## T:K:C NA.
```